Operations/functions defining algebraic/arithmetic expressions

Solutions.

1. I. $z\,\theta\,(-z)=\frac{1}{z}-\frac{1}{z}=0$ --> true.

   II. $z\,\theta\,\frac{z}{(z-1)}=\frac{1}{z}+\frac{z-1}{z}=1$ --> true.

   III. $\frac{2}{z}\,\theta\,\frac{2}{z}=\frac{z}{2}+\frac{z}{2}=z$ --> true.

   Answer: E.

2. $(5@45)@60=(\sqrt{5*45})@60=(\sqrt{225})@60=15@60=\sqrt{15*60}=30$.

   Answer: A.

3. Given: $V=\frac{1}{(2r)^3}=\frac{1}{8r^3}$

   Now, halved r is r/2, substitute r/2 instead of r: $V'=\frac{1}{(2*\frac{r}{2})^3}=\frac{1}{r^3}$.

   $V=\frac{1}{8r^3}$ should be multiplied by 8 to get $V'=\frac{1}{r^3}$.

   Answer: B.

   Or:

   When r=2, then $V=\frac{1}{(2r)^3}=\frac{1}{64}$.

   When r=1, then $V'=\frac{1}{(2r)^3}=\frac{1}{8}$.

   1/64 should be multiplied by 8 to get 1/8.

   Answer: B.

4. We have that: $a@b=a+b-ab$

I. $a@b=b@a$ --> $a@b=a+b-ab$ and $b@a=b+a-ab$ --> $a+b-ab=b+a-ab$, results match;

II. $a@0=a$ --> $a@0=a+0-a*0=0$ --> $0=0$, results match;

III. $(a@b)@c=a@(b@c)$ -->
$(a@b)@c=a@b+c-(a@b)*c=(a+b-ab)+c-(a+b-ab)c=a+b+c-ab-ac-bc+abc$ and
$a@(b@c)=a+b@c-a*(b@c)=a+(b+c-bc)-(b+c-bc)a=a+b+c-bc-ab+abc$, results match.

Answer: E.

5. Given that $a@b=\frac{a-b}{a+b}$, thus $a@c=\frac{a-c}{a+c}$.

Also given that $a@c=\frac{a-c}{a+c}=0$ --> $a-c=0$ --> $a=c$.

Answer: E.

6. Since $f(x,y)=x^2-y$, then $f(3,b)=9-b$ and $f(a,3)=a^2-3$. The question asks whether $9-b<a^2-3$ --> is $a^2+b>12$?

(1) a < b. If $a=1$ and $b=2$, then the answer is NO but $a=3$ and $b=4$, then the answer is YES. Not sufficient.

(2) a + 4 < 0 --> $a<-4$. If $a=-5$ and $b=0$, then the answer is YES but $a=-5$ and $b=-15$, then the answer is NO. Not sufficient.

(1)+(2) From (1) we have that $a<b$. Add a^2 to both parts: $a^2+a<a^2+b$. Now, since $a<-4$, then $a^2+a>12$ (if a=-4 then a^2+a=12 and if we decrease a, then we increase a^2+a). Thus we have that $12<a^2+a<a^2+b$. Sufficient.

Answer: C.

7. Given: $Q=\frac{5w}{4x*z^2}$.

Now, quadruple w, so make it $4w$; double x so make it $2x$; triple z and substitute these values instead of x, y, and z in the original equation:

. Thus Q is multiplied by $\frac{2}{9}$.

Answer: B.

Else plug-in values for x, y, and z. Let $x=y=z=1$ --> $Q=\frac{5w}{4x*z^2}=\frac{5}{4}$.

$4w=4, 2x=2$ and $3z=3$ --> $\frac{5*4}{4*2*3^2}=\frac{4}{18}*\frac{5}{4}=\frac{2}{9}*\frac{5}{4}$. Thus Q is multiplied by $\frac{2}{9}$.

Answer: B.

8. **Couple of things before solving:**

If exponentiation is indicated by stacked symbols, the rule is to work from the top down, thus:
$a^{m^n}=a^{(m^n)}$ and not $(a^m)^n$, which on the other hand equals to a^{mn}.

So:
$(a^m)^n=a^{mn}$;

$a^{m^n}=a^{(m^n)}$ and not $(a^m)^n$.

Back to the original question:
Let's replace # by @ as # looks like the symbol "not equal to" and it might confuse someone.

Given: $x@n=x^{(x@(n-1))}$ and $x@1=x$:
____

$x@2=x^{(x@1)}=x^x$, as $x@1=x$;
$x@3=x^{(x@2)}=x^{(x^x)}=x^{x^x}$;
$x@4=x^{(x@3)}=x^{(x^{x^x})}=x^{x^{x^x}}$;
...
Basically n in x@n represents the # of stacked x-es.

A. $(3@2)@2=(3^3)@2=(27)@2=27^{27}=3^{81}$

B. $3@(1@3)=3@(1^{1^1})=3@1=3$

C. $(2@3)@2=(2^{2^2})@2=16@2=16^{16}=2^{64}$

D. $2@(2@3)=2@(2^{2^2})=2@16$ this will be huge number 2^2^2^2^2^2^2^2^2^2^2^2^2^2^2^2=2^2^2^2^2^2^2^2^2^2^2^2^4=2^2^2^2^2^2^2^2^2^2^2^2^16=.... ;

E. $(2@2)@3=(2^2)@3=4@3=4^{4^4}=4^{256}=2^{512}$

Option D (2^2^2^2^2^2^2^2^2^2^2^2^2^2^2^2) will be much bigger number than numbers from other answer choices.

Answer: D.

9. Some function (#) is defined for all numbers a and b as $a\#b=a+b-ab$.

Now, since given that $a\#b=0$, then $a+b-ab=0$ --> $a=\frac{b}{b-1}$ --> if $b=1$ then the given expression is undefined so b cannot equal to 1.

Or: $a+b-ab=0$ --> $(a-1)(1-b)+1=0$ --> $(a-1)(1-b)=-1$. If $b=1$, then $(a-1)(1-b)=0$ not -1, so b cannot equal to 1.

Answer: B.

10. Since $s*t=(s-1)(t+1)$, then $(-2)*x=(-2-1)(x+1)=-3(x+1)$.

So, we are given that $-3(x+1)=-12$ --> $x+1=4$ -> $x=3$.

Answer: B.

11.

$$1\#(1\#a)=1\#(1+\frac{1}{a})=1\#(\frac{a+1}{a})=1+\frac{1}{(\frac{a+1}{a})}=1+\frac{a}{a+1}=\frac{a+1+a}{a+1}=\frac{2a+1}{a+1}$$

Answer: E.

12. a&b = (a - 1)(b - 1)
x&13 = (x-1)(13-1) = 96 --> x-1=8 --> x=9.

Answer: C.

13. (2 ^ ^ − 1) = $\frac{2^{2*(-1)}}{2}=-2$;

(−2 ^ ^1) = $\frac{(-2)^{2*1}}{2}=2$;

Thus, -2 ^ ^ 2 = $\frac{(-2)^{2*2}}{2}=4$.

Answer: D.

14. Any multiple of 6 is even.
Any two-digit prime number is odd.

(even+odd)/2 is not an integer. Thus # does not yield an integer at all.

Therefore P=0.

Answer: A.

15. Let's check each option:

I. $x?y=xy$ --> $y?x=yx$ --> $x?y-y?x=xy-yx=0$. Hence this option is ALWAYS equal to zero. Discard.

II. $x?y=(x-y)^2$ --> $y?x=(x-y)^2=(y-x)^2=(x-y)^2$ --> $x?y-y?x=(x-y)^2-(x-y)^2=0$. Hence this option is ALWAYS equal to zero. Discard.

III. $x?y=x^3+3x^2*y+3xy^2+y^3=(x+y)^3$ --> $y?x=(y+x)^3$ --> $x?y-y?x=(x+y)^3-(x+y)^3=0$. Hence this option is ALWAYS equal to zero. Discard.

Answer: ALL of the options equal to zero for ANY value of x and y. No correct answer among the answer choices.

If III were: $x?y=x^3-3x^2*y+3xy^2-y^3=(x-y)^3$, then $y?x=(y-x)^3$ --

> $x?y-y?x=(x-y)^3-(y-x)^3=(x-y)^3+(x+y)^3=2(x-y)^3$ --> this option equal to zero only if $x=y$.

So, in this case the answer would be C (III only)

16. Given: $a@b=\frac{a}{b}-\frac{b}{a}$, for all nonzero numbers a and b.

I. $x@(xy)=x(1@y)$: $LHS=x@(xy)=\frac{x}{xy}-\frac{xy}{x}=\frac{1}{y}-y$ and $RHS x(1@y)=x(\frac{1}{y}-y)$ as you see LHS doen't equal to RHS;

II. $x@y=-(y@x)$: $LHS=x@y=\frac{x}{y}-\frac{y}{x}$ and $RHS=-(y@x)=-(\frac{y}{x}-\frac{x}{y})=\frac{x}{y}-\frac{y}{x}$ --> LHS=RHS;

III. $(\frac{1}{x})@(\frac{1}{y})=y@x$: $LHS=(\frac{1}{x})@(\frac{1}{y})=\frac{y}{x}-\frac{x}{y}$ and $RHS=y@x=\frac{y}{x}-\frac{x}{y}$ --> LHS=RHS.

Answer: E (II and III).

17. @(15)=14. We have to compute @(...@(@(@(14)))...) 98 times. Each @ just adds 2 to the previous result. Therefore, @(...@(@(@(14)))...)98 times=14+2∗98=210.

Answer: C.

Consider this: @(@(@14)) = @(@16) = @18 = 20 = 14+2*3 (for 3 @'s). So, for 98 @ we'll have 14+2*98=210.

18. As k* denotes **the product** of all the fractions of the form **1/t**, where t is an integer between 1 and k, **inclusive** then $5*=\frac{1}{1}*\frac{1}{2}*\frac{1}{3}*\frac{1}{4}*\frac{1}{5}=\frac{1}{5!}$ and $4*=\frac{1}{1}*\frac{1}{2}*\frac{1}{3}*\frac{1}{4}=\frac{1}{4!}$ --> $\frac{5*}{4*}=\frac{4!}{5!}=\frac{1}{5}$.

Answer: E.

19. $f(x) = x^x$

$$f[f(x)] = f(x)^{f(x)}$$

$$= (x^x)^{(x^x)}$$

$$= x^{(x * x^x)}$$

$$= x^{x^{1+x}}$$

= Answer = D

20. 1) t Δ 2 = 74 --> (t + 2)^2 + (2 + 3)^2 = 74 --> t^2 + 4t - 45 = 0 --> t = 5 OR t = -9. Two values for t. Not sufficient.

(2) 2 Δ t = 80 --> (2 + 2)^2 + (t + 3)^2 = 80 --> t^2 +6t - 55 = 0 --> t = 5 OR t = -11. Two values for t. Not sufficient.

(1)+(2) t=5. Sufficient.

Answer: C.